PAPER ID-411139

Roll No: | | | | | | | | | | | | | |

BTECH
(SEM IV) THEORY EXAMINATION 2023-24
MATHEMATICS –IV

TIME: 3 HRS M.MARKS: 70

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 7 = 14

a	Solve the Partial Differential Equation $(D^2-3DD')z=0$	
	आंशिक अंतर समीकरण हल करें $(D^2-3DD')z=0$	
b	Tell the classification of the following partial differential equation $4\frac{\partial^2 u}{\partial x^2}-4\frac{\partial^2 u}{\partial x \partial t}+\frac{\partial^2 u}{\partial t^2}=0$	$4\frac{\partial^2 u}{\partial x^2}-4\frac{\partial^2 u}{\partial x \partial t}+\frac{\partial^2 u}{\partial t^2}=0$
	निम्नलिखित आंशिक अवकल समीकरण का वर्गीकरण बताइय $4\frac{\partial^2 u}{\partial x^2}-4\frac{\partial^2 u}{\partial x \partial t}+\frac{\partial^2 u}{\partial t^2}=0$	
c	Write the formula for rank correlation in the case of tied ranks.	
	बराबर रैंक क मामल में रैंक सहसंबंध का सूत्र लिखें।	
d	Calculate the value of k for which function $f(x)=\begin{cases}\frac{1}{k} & a\le x\le b\\ 0 & \text{otherwise}\end{cases}$ is probability density function.	
	किस फ़ंक्शन कलिए k का मान परिकलित करें $f(x)=\begin{cases}\frac{1}{k} & a\le x\le b\\ 0 & \text{otherwise}\end{cases}$ संभाव्यता घनत्व फ़ंक्शन ह	
e	Two coins are tossed simultaneously. What is the the probability (upto two decimal points accuracy) of getting at least one head .	
	दो सिक्क एक साथ उछाल जात हैं। कम स कम एक चित आन की प्रायिकता (दो दशमलव अंकों तक सटीकता) क्या ह?	
f	For the Binomial distribution $\left(\frac{1}{3}+\frac{2}{3}\right)^{10}$ determine the mean.	
	द्विपद वितरण कलिए $\left(\frac{1}{3}+\frac{2}{3}\right)^{10}$ माध्य निर्धारित करें.	
g	Write the control limits of C charts.	
	सी चार्ट की नियंत्रण सीमाएँ लिखें।	

SECTION B

2. Attempt any *three* of the following: 7 x 3 = 21

a	Determine a real function V of x and y, reducing to zero when y=0 and satisfying $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=-4x(x^2+y^2)$	$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=-4x(x^2+y^2)$
	x और y का एक वास्तविक फ़ंक्शन V निर्धारित करें, जो y=0 होन पर शून्य हो जाए और संतुष्ट हो $\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 v}{\partial y^2}=-4x^2-y^2$	

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b	A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, Evaluate the displacement $y(x, t)$.
	निश्चित अंत बिंदु x=0 और x=l का साथ एक कसकर खींची गई स्ट्रिंग प्रारंभ में निम्न स्थिति में ह $y = y_0 \sin^3 \frac{\pi x}{l}$ यदि इसाइस स्थिति साविराम साछोड़ा जाए तो विस्थापन y(x, t) का मूल्यांकन करें।
c	From the data given, determine the lines of regression. दिए गए डटा सा प्रतिगमन की रखाएं निर्धारित करें

x	2	4	6	8	10
y	5	7	9	8	11

d	Prove that Poisson distribution is as a limiting case of binomial distribution.
	सिद्ध कीजिए कि पॉइसन वितरण द्विपद वितरण का एक सीमित मामला ह।
e	Distinguish between np-chart and p-chart. Following is the data of defective of 10 samples of size 100 each. Construct np-chart and explain your findings. np-चार्ट और p-चार्ट का बीच अंतर बताइए। नीच 100 आकार का 10 नमूनों का दोषपूर्ण होना का डटा दिया गया ह। np-चार्ट बनाएँ और अपन निष्कर्षों की व्याख्या करें।

Sample no.	1	2	3	4	5	6	7	8	9	10
No. of defectives	6	9	12	5	12	8	8	16	13	7

SECTION C

3. Attempt any *one* part of the following: 7 x 1 = 7

(a)	Solve the partial differential equation $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = \log x$
	आंशिक अंतर समीकरण हल करें $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = \log x$
(b)	By using Charpit's method to evaluate the solution of $2zx - px^2 - 2qxy + pq = 0$
	$2zx - px^2 - 2qxy + pq = 0$ का समाधान का मूल्यांकन करना के लिए चार्पिट की विधि का उपयोग करा

4. Attempt any *one* part of the following: 7 x 1 = 7

(a)	Determine the Fourier Transform of $f(x) = \begin{cases} 1 - x^2 & x < 1 \\ 0 & x > 1 \end{cases}$

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	$f(x) = \begin{cases} 1 - x^2 & \lvert x \rvert \le 1 \\ 0 & x > 1 \end{cases}$ का फूरियर रूपांतरण निर्धारित करें
(b)	An insulated rod of length / has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C then determine the temperature at a distance x from A at time t.
	/ लंबाई की एक इन्सुलेटेड छड़ के सिरे A और B को क्रमशः 0°C और 100°C पर स्थिर अवस्था की स्थिति बननेतक बनाए रखा जाता है। यदि B को अचानक 0°C तक कम कर दिया जाता है और 100°C पर बनाए रखा जाता है, तो समय t पर A से x दूरी पर तापमान निर्धारित करें।

5. Attempt any *one* part of the following: 7 x 1 = 7

(a) . Examine the least square fit of the $f(x) = a + bx + cx^2$ the following data:

x	0	1	2	3	4
f(x)	1	4	10	17	30

निम्नलिखित डेटा $f(x) = a + bx + cx^2$ के न्यूनतम वर्ग फिट की जाँच करें:

x	0	1	2	3	4
f(x)	1	4	10	17	30

(b) Determine the skewness and kurtosis for the following data:
निम्नलिखित डेटा के लिए तिरछापन और कुर्टोसिस निर्धारित करें

निशान (Marks)	10-20	20-30	30-40	40-50	50-60
छात्रों की संख्या (No. of students)	18	20	30	22	10

6. Attempt any *one* part of the following: 7 x 1 = 7

(a) Records taken of the number of male and female births in 800 families having four children are as follows:

No. of male birth	0	1	2	3	4
No. of female birth	4	3	2	1	0
No. of families	32	178	290	236	64

Test whether the data are consistent with the hypothesis that the binomial law holds and the chance of male birth is equal to that of female birth , namely p=q=1/2.

चार बच्चों वाले 800 परिवारों में जन्म लड़कों और लड़कियों की संख्या का रिकार्ड इस प्रकार है:

No. of male birth	0	1	2	3	4
No. of female birth	4	3	2	1	0
No. of families	32	178	290	236	64

परीक्षण करें कि क्या डेटा उस परिकल्पना का अनुरूप है जो द्विपद नियम को लागू करता है और पुरुष जन्म की संभावना महिला जन्म की संभावना के बराबर है अर्थात p=q=1/2.

(b) The income of a group of 10,000 persons was found to be normally distributed with mean 750 Rs. per month and standard deviation of 50 Rs. Show that, of this group, about 95% had

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	income exceeding 668 Rs and only 5% had income exceeding 832 Rs. Also Determine the lowest income among the richest 100.
	10,000 व्यक्तियों का एक समूह की आय सामान्य रूप स वितरित पाई गई जिसका औसत 750 रुपय प्रति माह और मानक विचलन 50 रुपयाथा। दिखाएँ कि इस समूह में सालगभग 95% की आय 668 रुपयासे अधिक थी और वसल 5% की आय 832 रुपयासेअधिक थी। सबसेअमीर 100 में सेसबसेकम आय का भी निर्धारण करें।

7. Attempt any *one* part of the following: 7 x 1 = 7

(a)	The height of 6 randomly chosen sailors in inches are 63, 65,68,69,71 and 72. Those of 9 randomly chosen soldiers are 61, 62,65,66,69,70,71,72 and 73. Test whether the sailors are on the average taller than soldiers.
	यादृच्छिक रूप स चुन गए 6 नाविकों की ऊंचाई इंच में ह। 63, 65,68,69,71 and 72. यादृच्छिक रूप स चुनेगए 9 सैनिकों में से61, 62,65,66,69,70,71,72 and 73. परीक्षण करें कि क्या नाविक औसतन सैनिकों स लम्ब होत हैं।
(b)	Discuss whether there is any association between Area and pollution index of the city for which the data is given

Area/Pollu.ind.	high	low	Total
Urban	150	350	500
Rural	250	350	600

Given that $\chi^2_{0.05} = 3.84$

चर्चा करें कि क्या शहर का क्षेत्र और प्रदूषण सूचकांक का बीच कोई संबंध ह।जिसका लिए डेटा दिया गया ह।

क्षेत्र/प्रदूषण उद्योग.	उच्च	कम	कुल
शहरी	150	350	500
ग्रामीण	250	350	600

मान लें कि $\chi^2_{0.05} = 3.84$.

Maths-4 BAS-403

Sem-IV. Mathematics-IV. (BAS-403)

Section-A

(1) $(D^2-3DD')z=0$

Sol For CF $m^2-3m=0 \Rightarrow m(m-3)=0 \Rightarrow m=0,3$ → 1 Mark

$z=f_1(y)+f_2(y+3x)$ → 1 Mark

(b) $4\frac{\partial^2 u}{\partial x^2}-4\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial^2 u}{\partial t^2}=0$

Sol $A=4,\ B=-4,\ C=1$

$B^2-4AC = 16-4\times4=0 \Rightarrow$

Hence the given partial differential equation is Parabolic → 2 Mark

(c) $r=1-\dfrac{6\left[\Sigma d^2+\frac{1}{12}m_1(m_1^2-1)+\frac{1}{12}m_2(m_2^2-1)+\cdots\right]}{n(n^2-1)}$ → 2 Mark

where m stands for the number of times an item is repeated

(d) For the Probability density function,

$$\int_{-\infty}^{\infty} f(x)\,dx=1 \Rightarrow \int_a^b \frac{1}{K}\,dx=1$$

$\Rightarrow \frac{1}{K}(b-a)=1 \Rightarrow \frac{1}{K}(b-a)=K \Rightarrow \boxed{K=(b-a)}$

→ 2 Mark

Sin (e) Two coin can be tossed $= \frac{1}{2^2} = \frac{1}{4}$

(e) The Probability of getting at least one head
$= 1 - \frac{1}{4} = \frac{3}{4}$ ⟶ 2 Mark

(f) $q = \frac{1}{3}$. $P = \frac{2}{3}$, $n = 10$
Mean $= nP = 10 \times \frac{2}{3} = \frac{20}{3}$ ⟶ 2 Mark

(g) Control Limits for C chart

$$UCL = \bar{C} + 3\sqrt{\bar{C}}$$
$$LCL = \bar{C} - 3\sqrt{\bar{C}}$$
$$CL = \bar{C}$$

⟶ 2 Mark

Section - B

2(a) $(D^2 + D'^2)V = -4\pi(x^2+y^2)$

Auxiliary eq^n is $m^2 + 1 = 0 \Rightarrow m^2 = -1 = i^2 \Rightarrow m = \pm i$

C.F $y = f_1(y+ix) + f_2(y-ix)$ ⟶ 3 Mark

$$\text{P.I} = \frac{1}{D^2 + D'^2}\left[-4\pi(x^2+y^2)\right] = -\frac{4\pi}{D^2}\left[1 + \frac{D'^2}{D^2}\right](x^2+y^2)$$

$$= -\frac{4\pi}{D^2}\left[1 - \frac{D'^2}{D^2}\right](x^2+y^2) = -\frac{4\pi}{D^2}\left[(x^2+y^2) - \frac{1}{D^2}(2)\right]$$

$$= -\frac{4\pi}{D^2}\left[x^2+y^2-x^2\right] = -\frac{4\pi}{D^2}(y^2) = -\frac{4\pi}{D}(xy^2) = -2\pi x$$

⟶ 3 Mark

Since V is real function hence $V = -2xy$ (3) → 1 Mark

3

(b) The equation of the string is $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ — (1)

The solution of eqn (1)

$$y(x,t) = (C_1 \cos cPt + C_2 \sin cPt)(C_3 \cos Px + C_4 \sin Px)$$

→ 2 Mark → (2)

Boundary Conditions are.

$y(0,t) = 0$ → (3)

$y(l,t) = 0$ → (4)

$\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$ → (5)

$y(x,0) = y_0 \sin^3 \frac{\pi x}{l}$ → (6)

Apply the Boundary condition in eqn (2):

$$y(0,t) = 0 = (C_1 \cos cPt + C_2 \sin cPt)\, C_3 \Rightarrow C_3 = 0$$

From eqn (2),

$$y(x,t) = (C_1 \cos cPt + C_2 \sin cPt)\, C_4 \sin Px \rightarrow (7)$$

$$y(l,t) = 0 = (C_1 \cos cPt + C_2 \sin cPt)\, C_2 \sin Pl$$

$\sin Pl = 0 = \sin n\pi \Rightarrow Pl = n\pi \Rightarrow \boxed{P = \frac{n\pi}{l}}$ → 2 Mark

From eqn (7)

$$y(x,t) = \left(C_1 \cos \frac{cn\pi}{l} t + C_2 \sin \frac{cn\pi}{l} t\right) C_4 \cos \frac{n\pi}{l} x \rightarrow (8)$$

$$\frac{\partial y}{\partial t} = \frac{n\pi c}{l}\left[-c_1 \sin\frac{n\pi ct}{l} + c_2 \cos\frac{n\pi ct}{l}\right]\left[c_4 \sin\frac{n\pi x}{l}\right] \quad (4)$$

At $t=0$

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = \frac{n\pi c}{l}\left[c_2\right]\left[c_4 \sin\frac{n\pi x}{l}\right]$$

Given $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0 \Rightarrow \frac{n\pi c}{l} c_2 c_4 \sin\frac{n\pi x}{l} = 0 \Rightarrow \boxed{c_2 = 0}$

$$\therefore y(x,t) = c_1 c_4 \cos\frac{cn\pi}{l}t \sin\frac{n\pi}{l}x \longrightarrow (9)$$

Given. $y = y_0 \sin^3\frac{\pi x}{l} \longrightarrow (*)$

$$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\sin^3\theta = \left(\frac{3\sin\theta - \sin 3\theta}{4}\right)$$

The General Solution is. $\longrightarrow$ 1 Mark

$$y(x,t) = \sum_{n=1}^{\infty} b_n \cos\frac{cn\pi}{l}t \sin\frac{n\pi}{l}x \longrightarrow (10)$$

Given. $y(x,0) = y_0 \sin^3\frac{\pi x}{l} \longrightarrow (*1)$

$$y(x,0) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{l}x \longrightarrow (11)$$

$$y_0 \sin^3\frac{\pi x}{l} = b_1 \sin\frac{\pi x}{l} + b_2 \sin\frac{2\pi}{l}x + b_3 \sin\frac{3\pi x}{l}$$

$$\frac{y_0}{\therefore}\left[3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}\right] = b_1 \sin\frac{\pi x}{l} + b_2 \sin\frac{2\pi x}{l} + b_3 \sin\frac{3\pi}{l}$$

~~Scatter diagram~~ Chart

6

2 (c) mid

x	y	x^2	y^2	xy
2	5	4	25	10
4	7	16	49	28
6	9	36	81	54
8	8	64	64	64
10	11	100	121	110
$\Sigma x = 30$	$\Sigma y = 40$	$\Sigma x^2 =$ ~~220~~	$\Sigma y^2 =$ 340	$\Sigma xy = 266$

→ 3 Mark

$$b_{yx} = \frac{5 \times 266 - (30) \times (40)}{5 \times \text{~~220~~} - (30)^2} = \frac{1330 - 1200}{600 - 900} = \frac{130}{200} = 0.65$$

$$b_{xy} = \frac{5 \times 266 - (30) \times 40}{5 \times 340 - (40)^2} = \frac{130}{1700 - 1600} = \frac{130}{100} = 1.3$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{30}{5} = 6, \quad \bar{y} = \frac{40}{5} = 8$$

Eq^n of Regression Line y on x

$$y - 8 = \text{~~0.43~~ } 0.65 (x - 6)$$

$$y - 8 = 0.65x - 3.9$$

$$y = 0.65x + 8 - 3.9$$

$$\boxed{y = 0.65x + 4.1}$$

Similarly the Eq^n of Regression line x on y

$$(x - \bar{x}) = b_{xy} (y - \bar{y})$$

$$x - 6 = 1.3 (y - 8)$$

$$x = 1.3y + 6 - 10.4$$

$$\boxed{x = 1.3y - 4.4}$$

→ 2 Mark

~~Comparing the coefficients of both sides~~

& (b) end of

$b_1 = \frac{3y_0}{4}$, $b_2 = 0$, $b_3 = -\frac{y_0}{4}$, $b_4 = b_5 = b_6 \cdots = 0$

Thus the solution is

$$y(x,t) = b_1 \cos\frac{c\pi t}{l} \sin\frac{\pi x}{l} + b_3 \cos\frac{c3\pi t}{l} \sin\frac{3\pi x}{l}$$

$$y(x,t) = \frac{3y_0}{4} \cos\frac{c\pi t}{l} \sin\frac{\pi x}{l} - \frac{y_0}{4} \cos\frac{3c\pi t}{l} \sin\frac{3\pi x}{l}$$

→ 2 Mark Ans.

(c) The Eqⁿ of Lines of Regression are given by

2C

Starting $(y - \bar{y}) = b_{yx}(x - \bar{x})$ → (1)

$(x - \bar{x}) = b_{xy}(y - \bar{y})$ → (2)

→ 2 Mark

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \rightarrow (3)$$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2} \rightarrow (4).$$

(1) Poisson Distribution is a Limiting case of Binomial Distribution.

The Mean of the Binomial Distribution

$$P(X=r) = {}^nC_r\, p^r q^{n-r} \longrightarrow (1)$$

$$= {}^nC_r\, p^r (1-p)^{n-r} \longrightarrow (2)$$

$$= \frac{\lfloor n}{\lfloor r \lfloor n-r} p^r (1-p)^{n-r} \longrightarrow \text{2 Mark.}$$

Let $\lambda = np \Rightarrow p = \frac{\lambda}{n}$.

$$= \frac{\lfloor n}{\lfloor r \lfloor n-r} \left(\frac{\lambda}{n}\right)^r \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{n(n-1)(n-2)\cdots(3.2.1)}{\lfloor r \lfloor n-r} \left(\frac{\lambda}{n}\right)^r \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{n(n-1)(n-2)\cdots n-(r-1)(n-r)\cdots 3.2.1}{\lfloor r \lfloor n-r} \left(\frac{\lambda}{n}\right)^r \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{n(n-1)(n-2)\cdots (n-(r)+1)\lfloor n-r}{\lfloor r \lfloor n-r} \frac{\lambda^r}{n^r} \left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$= \left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\cdots\left(\frac{n-r+1}{n}\right) \frac{\lambda^r}{\lfloor r} \left(1-\frac{\lambda}{n}\right)^n \left(1-\frac{\lambda}{n}\right)^{-r}$$

$\longrightarrow$ 2 Mark

$$P(X=r) = \frac{1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left[1-\frac{(r-1)}{n}\right]}{\lfloor r} \lambda^r \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r} \quad (8)$$

Taking the limit $n \to \infty$.

$$= \lim_{n\to\infty} \frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots 1-\frac{(r-1)}{n}\right]}{\lfloor r} \lambda^r \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$\because \lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda}$$

→ 3 Mark.

$$\boxed{P(X=r) = \frac{\lambda^r e^{-\lambda}}{\lfloor r}}$$

(c) Difference between P chart and np chart

P chart show the proportion of defective units on the y axis.
NP chart show the whole number of defective units on y axis. → 1 Mark

Total No of defectives = 96

Σn = total number inspected = 100×10

$$\bar{P} = \frac{\Sigma np}{\Sigma n} = \frac{96}{1000} = 0.096 \longrightarrow 1 \text{ Mark}$$

$$n\bar{P} = 100 \times 0.096 = 9.600$$

$$CL = n\bar{P} = 9.6$$

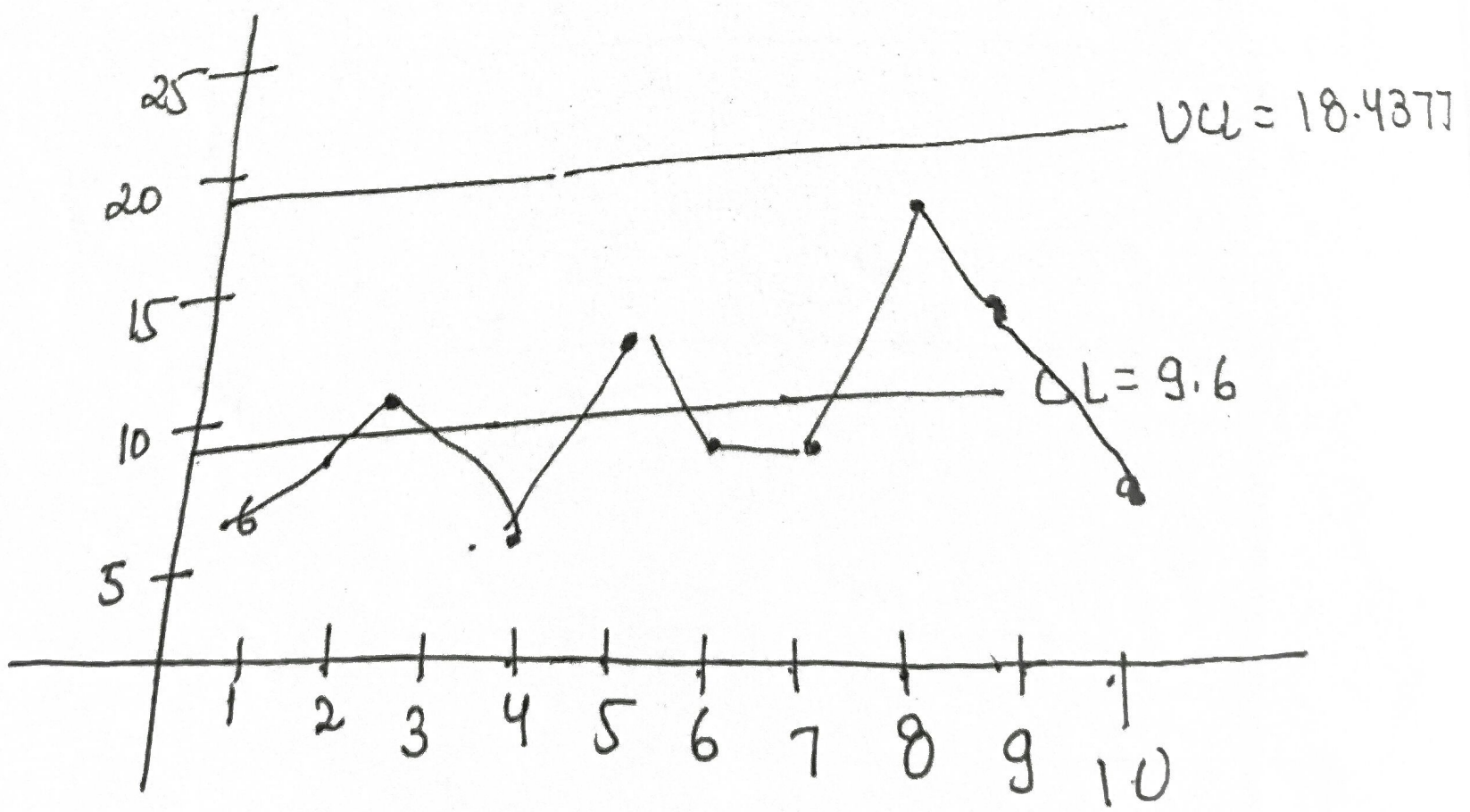
$$UCL = n\bar{P} + 3\sqrt{n\bar{P}(1-\bar{P})} = 9.6 + 3\sqrt{9.6(1-0.096)}$$

$$= 18.4377$$

$$LCL = n\bar{P} - 3\sqrt{n\bar{P}(1-\bar{P})} = 9.6 - 3\sqrt{9.6(1-0.096)}$$

$$= 0.7622$$

2 Mark



2 Mark

$2\Delta')(\Delta - 2\Delta') + 2\Delta' z = e^{3\lambda} e^{4\lambda}$

Since all the points lie within the control limits the statistical process is under control. (10)

→ 1 Mark

Section - C

3(a) $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \log x$

Sol Let $x = e^X$, $y = e^Y \Rightarrow X = \log x$, $Y = \log y$

$D = \frac{\partial}{\partial X}$, $D' = \frac{\partial}{\partial Y}$, $DD' = \frac{\partial^2}{\partial X \partial Y}$

$$\left[D(D-1) - D'(D'-1) + D - D'\right] z = X$$

$$\left[D^2 - D'^2\right] z = X$$

which is a homogenous linear Pde with constant coefficients.

$$CF = \phi_1(Y+X) + \phi_2(Y-X) = \phi_1(\log y + \log x) + \phi_2(\log y - \log x)$$ ⟶ 3 Mark

$$P.I = \frac{1}{D^2 - D'^2} X = \frac{1}{(1)^2 - (0)^2}\iint u\, du\, du$$

$$= \int \frac{u^2}{2}\, du = \frac{(\log x)^3}{6} = \frac{X^3}{6}$$ ⟶ 2 Mark

$$z = \phi_1(\log y + \log x) + \phi_2(\log y - \log x) + \frac{(\log x)^3}{6}$$ Ans

⟶ 2 Mark.

3(b) $f \equiv 2zx - px^2 - 2qxy + pq = 0$

$\frac{\partial f}{\partial x} = 2z - 2px - 2qy$; $\frac{\partial f}{\partial y} = -2qy$, $\frac{\partial f}{\partial z} = 2x$, $\frac{\partial f}{\partial p} = -x^2 + q$

$\frac{\partial f}{\partial q} = -2xy + p$ ⟶ 2 Mark

Charpit's auxiliary equation are

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{df}{0}$$

⟶ 1 Mark

$$\frac{dp}{2z - 2qy} = \frac{dq}{0} = \frac{dz}{px^2 - 2pq + 2qxy} = \frac{dx}{x^2 - q} = \frac{dy}{2xy - p}$$

$dq = 0 \Rightarrow q = a$

Putting $q = a$ in (1), $p = \frac{2x(z - ay)}{x^2 - a}$

$dz = p\,dx + q\,dy$

$dz = \frac{2x(z - ay)}{x^2 - a}dx + a\,dy$ ⟶ 3 Mark

$$\frac{dz - a\,dy}{z - ay} = \frac{2x}{x^2 - a}dx$$

$\log(z - ay) = \log(x^2 - a) + \log b$.

$(z - ay) = b(x^2 - a)$ ⟶ 1 Mark

$(\log y + \log x) + x\,\phi_2(\log$

$f(x)$ $f(x) = \begin{cases} (1-x) & x<1 \\ 0 & x>1 \end{cases}$

(13)

Sol The fourier transform of $f(x)$ is given by.

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2)\, e^{-isx}\, dx + 0 \longrightarrow$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2)(\cos sx - i \sin sx)\, dx.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) \cos sx\, dx - \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) \sin sx\, dx$$

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{1} (1-x^2) \cos sx\, dx = \sqrt{\frac{2}{\pi}} \left[(1-x^2)\left(\frac{\sin sx}{s}\right)_0^1 - \int_0^1 (-2x)\left(\frac{\sin sx}{s}\right) \right.$$

$$= \frac{2}{s}\sqrt{\frac{2}{\pi}} \int_{0}^{1} x \sin sx\, dx \longrightarrow 4 \text{ Mark}$$

$$= \frac{2}{s}\sqrt{\frac{2}{\pi}} \left[-\left(\frac{x \cos sx}{s}\right)_0^1 - \int_0^1 -\frac{\cos sx}{s}\, dx \right]$$

$$= \frac{2}{s}\sqrt{\frac{2}{\pi}} \left[-\frac{\cos s}{s} + \left(\frac{\sin sx}{s^2}\right)_0^1 \right]$$

$$= -\frac{2}{s^3}\sqrt{\frac{2}{\pi}} (s\cos s - \sin s) \longrightarrow 1 \text{ Mark}$$

Initial temperature distribution in the rod is

$$u_1 = 0 + \left(\frac{100-0}{l}\right)x = \frac{100x}{l} \qquad \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \rightarrow (1)$$

final temperature distribution

$$u_2 = 0 + \left(\frac{0-0}{l}\right)x = 0$$

2 Mark

$$u = u_2(x) + u_1(x,t)$$

$$u(x,t) = \sum_{n=1}^{\infty} (a_n \cos Px + b_n \sin Px)\, e^{-c^2p^2t} \rightarrow (2)$$

$$u(0,t) = 0 = u(l,t)$$

$$u(0,t) = 0 = \sum_{n=1}^{\infty} a_n e^{-c^2p^2t} \Rightarrow \boxed{a_n = 0}$$

$$u(l,t) = \sum_{n=1}^{\infty} b_n \sin Pl\; e^{-c^2p^2t} = 0$$

$$\sin Pl = 0 = \sin n\pi \Rightarrow \boxed{P = \frac{n\pi}{l}}$$

2 Mark

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}\, e^{-\frac{c^2 n^2 \pi^2}{l^2}t} \rightarrow (3)$$

$$u(x,0) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$\frac{100x}{l} = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$b_n = \frac{2}{l}\int_0^l \frac{100x}{l} \sin\frac{n\pi x}{l}\, dx \longrightarrow 1 \text{ Mark}$$

$$= \frac{2}{l} \times \frac{100}{l}\left[-x\cos\frac{n\pi x}{l}\,\frac{l}{n\pi}\right]_0^l - \int_0^l 1\cdot\left(-\cos\frac{n\pi x}{l}\right)\frac{l}{n\pi}\,dx\Bigg]$$

$$= -\frac{200}{n\pi}(-1)^n \longrightarrow 1 \text{ Mark}$$

$= \phi_1(\log y + \log x) + x\,\phi_2(\log y - \log x)$

$u(x,t) = \frac{-200}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\frac{n\pi x}{l}\, e^{-\frac{n^2\pi^2c^2}{l^2}t}$ (15)

$4D^2D' + 2D'$

$\mu_1' = \Sigma$

$\mu_2' = \Sigma$

$\mu_3' = \Sigma$

$\mu_4' = \Sigma$

Momen

$\mu_1 =$

μ_2

μ_3

μ

e^{x+y}

$D'^2 - D + D$

case

$=$

$\frac{x}{x1}$ ϕ

gx x

$(\log$

$+ l$

eg

ue

→ 1 Mark.

5(a)

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	4	1	1	1	4	4
2	10	4	8	16	20	40
3	17	9	27	81	57	153
4	30	16	64	256	120	480
$\Sigma x=10$	$\Sigma y=62$	$\Sigma x^2=30$	$\Sigma x^3=100$	$\Sigma x^4=354$	$\Sigma xy=195$	$\Sigma x^2y=677$

→ 3 Mark

Let $y = a + bx + cx^2$ → (1)

$\Sigma y = na + b\Sigma x + c\Sigma x^2$ → (2)

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$ → (3)

$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ → (4)

$62 = 5a + 10b + 30c$ → (5)

$195 = 10a + 30b + 100c$ → (6)

$677 = 30a + 100b + 354c$ → (7)

→ 2 Mark

Solving eqn (5), (6), (7) $a = 1.2,\ b = 1.1,\ c = 1.5$ → 1 Mark

$y = 1.2 + 1.1x + 1.5x^2$ Ans → 1 Mark

5(b)

class	x	f	$u = \frac{x-35}{10}$	fu	fu^2	fu^3	fu^4
10-20	15	18	-2	-36	72	-144	288
20-30	25	20	-1	-20	20	-20	20
30-40	35	30	0	0	0	0	0
40-50	45	22	1	22	22	22	22
50-60	55	10	2	20	40	80	16
				$\Sigma fu = -14$	$\Sigma fu^2 = 154$	$\Sigma fu^3 = \Sigma fu^4 =$	

2 Mark = 62

(15)

(16)

$$\mu_1' = \frac{\Sigma fu}{N} \times h = \frac{-14}{100} \times 10 = -1.4$$

$$\mu_2' = \frac{\Sigma fu^2}{N} \times h^2 = \frac{154}{100} \times 100 = 154$$

$$\mu_3' = \frac{\Sigma fu^3}{N} \times h^3 = \frac{-62}{100} \times 1000 = -620$$

$$\mu_4' = \frac{\Sigma fu^4}{N} \times h^4 = \frac{346}{100} \times 100 \times 100 = 346$$

→ 2 Mark.

Moments about Mean

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2 = 154$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= -620 - 3 \times 154 \times (-1.4) + 2(-1.4)^3$$

$$= -620 + 3 \times 154 \times 1.4 - 2 \times (1.4)^3 = 24.056$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 346 - 4 \times (-620) \times (-1.4) + 6 \times 154 \times (-1.4)^2 - 3 \times (-1.4)^4$$

$$= 1649.4352$$

→ 2 Mark

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{24.056 \times 24.056}{154 \times 154 \times 154} = \frac{578.691136}{3652264} = .000\,15845$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{1649.4352}{154 \times 154} = \frac{1649.4352}{23716} = .06954$$

$= \phi_1(\log y + \log x) + x\,\phi_2(\log y$

17

5(a)

	x	y	x^2	x^3	x^4	xy	x^2y
	0	1	0	0	0	0	0
	1	4	1	1	1	4	4
	2	10	4	8	16	20	40
	3	17	9	27	81	57	153
	4	30	16	64	256	120	480
	$\Sigma x=10$	$\Sigma y=62$	$\Sigma x^2=30$	$\Sigma x^3=100$	$\Sigma x^4=354$	$\Sigma xy=195$	$\Sigma x^2y=677$

→ 3 Marks

Let $y = a+bx+cx^2$ → (1)

$\Sigma y = na+b\Sigma x+c\Sigma x^2$ → (2)

$\Sigma xy = a\Sigma x+b\Sigma x^2+c\Sigma x^3$ → (3)

$\Sigma x^2y = a\Sigma x^2+b\Sigma x^3+c\Sigma x^4$ → (4)

→ 2 Mark

$62 = 5a+10b+30c$ → (5)

$195 = 10a+30b+100c$ → (6)

$677 = 30a+100b+354c$ → (7)

Solving eqⁿ (5), (6), (7) $a=1.2,\ b=1.1,\ c=1.5$ → 1 Mark

$y = 1.2+1.1x+1.5x^2$ Ans → 1 Mark

5(b)

class	x	f	$u=\frac{x-35}{10}$	fu	fu^2	fu^3	fu^4
10-20	15	18	-2	-36	72	-144	288
20-30	25	20	-1	-20	20	-20	20
30-40	35	30	0	0	0	0	0
40-50	45	22	1	22	22	22	22
50-60	55	10	2	20	40	80	16

$\Sigma fu = -14$ $\Sigma fu^2 = 154$ $\Sigma fu^3 = \Sigma fu^4 = 346$

$= e^{3x} e^{\cdot}$

(17)

$\gamma_1 > 0 \Rightarrow$ positively skewed curve
$\beta_1 < 3 \Rightarrow$ distribution is Platykurtic] 1 Mark

(19) (18)

Null Hypothesis : Chance of Male birth is not equal to that of female birth

(i) $p = \frac{1}{2}, \quad q = \frac{1}{2}$ $\quad P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

Here $n = 4$

$P(X = r) = {}^{4}C_{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{4-r} = {}^{4}C_{r} \left(\frac{1}{2}\right)^{4} \rightarrow (*)$

No of Birth	$P(X=r) = {}^{4}C_{r}\left(\frac{1}{2}\right)^{4}$	Expected frequency $N \times P(r)$
0	${}^{4}C_{0}\left(\frac{1}{2}\right)^{4} = \left(\frac{1}{2}\right)^{4}$	$800 \times \frac{1}{16} = 50$
1	${}^{4}C_{1}\left(\frac{1}{2}\right)^{4} = \frac{4}{16}$	$800 \times \frac{4}{16} = 200$
2	${}^{4}C_{2}\left(\frac{1}{2}\right)^{4} = \frac{6}{16}$	$800 \times \frac{6}{16} = 300$
3	${}^{4}C_{3}\left(\frac{1}{2}\right)^{4} = \frac{4}{16}$	$800 \times \frac{4}{16} = 200$
4	${}^{4}C_{4}\left(\frac{1}{2}\right)^{4} = \frac{1}{16}$	$800 \times \frac{1}{16} = 50$

→ 3 Mark

χ^2 Table.

O_i	E_i	$(O_i - E_i)$	$\frac{(O_i - E_i)^2}{E_i}$
32	50	−18	6.48
178	200	−22	2.42
290	300	−10	0.333
236	200	36	6.48
64	50	14	0.0784

$\sum \frac{(O_i - E_i)^2}{E_i} = 15.7914$

→ 2 Mark

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 15.7914$$

Degree of freedom = $n-1 = 4-1 = 3$

Level of significance = 5%.

$\chi^2_{0.005}(3) = 7.815$ ⟶ 1 Mark

$\because \chi^2_{cal} > \chi^2_{tab}$

⇒ Null Hypothesis is Rejected i.e Binomial law holds. ⟶ 1 Mark

6(b) $z = \frac{x-\mu}{\sigma}$

i) If $x_1 = 688$, $z_1 = \frac{x_1-\mu}{\sigma} = \frac{688-750}{50} = -1.64$ ⟶ 1 Mark

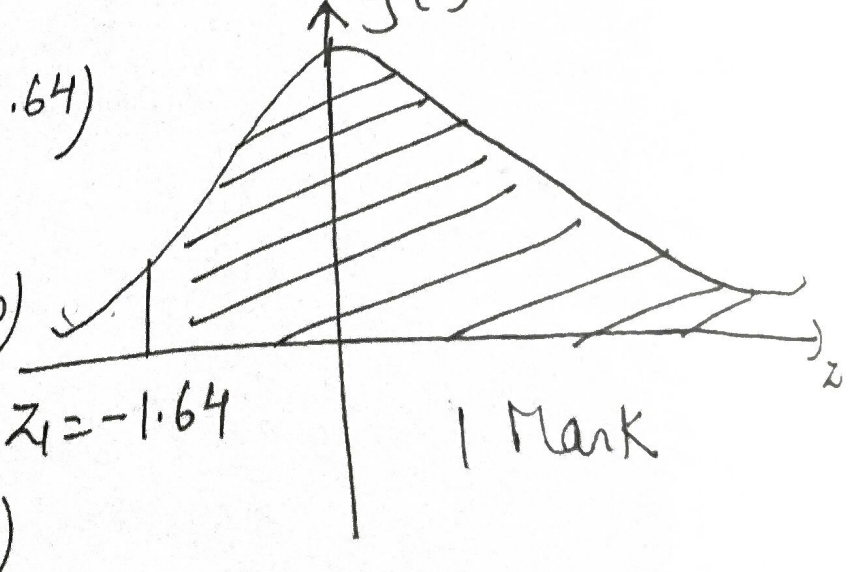
$P(x_1 > 668) = P(z_1 > -1.64)$

$= 0.5 + P(-1.64 \le z \le 0)$

$= 0.5 + P(0 \le z \le 1.64)$

$= 0.5 + 0.4495$

$= 0.9495$ ⟶ 1 Mark



1 Mark

Required Percentage of persons having income exceeding Rs 668 = 94.95%.

$D^2z - Dz - 4DD'z + 4D'z$

$(D^2 - Dz - 4DD' + 4D'^2 - 4D' + 6D')z = x^3y^4$

$(D^2 - 4DD' + 2D')z = x^3y^4$

19

$\therefore n-2$ ... 1.77

Null Hypothesis: Mean of both the Population are same → 1 Mark

20

X_1	63	65	68	69	71	72
$X_1 - \bar{X_1}$	-5	-3	0	1	3	4
$(X_1 - \bar{X_1})^2$	25	9	0	1	9	16

→ 1 Mark

$\bar{X_1} = \frac{\Sigma X_1}{n} = 68, \quad \Sigma(X_1 - \bar{X_1})^2 = 60$

X_2	61	62	65	66	69	70	71	72	73
$X_2 - \bar{X_2}$	-6.66	-5.66	-2.66	1.66	1.34	2.34	3.34	4.34	5.34
$(X_2 - \bar{X_2})^2$	44.36	32.035	7.0756	2.7556	1.796	5.4756	11.1556	18.8356	28.52

→ 1 Mark

$\bar{X_2} = \frac{\Sigma X_2}{n_2} = 67.66$

$\Sigma(X_2 - \bar{X_2})^2 = 152.0002$

$S^2 = \frac{1}{n_1 + n_2 - 2}\left[\Sigma(X_1 - \bar{X_1})^2 + \Sigma(X_2 - \bar{X_2})^2\right] = 16.3077$ → 1 Mark

$S = 4.038$

$t = \frac{\bar{X_1} - \bar{X_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{68 - 67.666}{4.038\sqrt{\frac{1}{6} + \frac{1}{9}}} = 0.1569$ → 1 Mark

The value of t at 5% level of significance for 13 d.f is 1.77

→ 1 Mark

...omial law

→ 1 Mark

-1.64

→ 1 Mark

$x) + e^x \phi_2(y-x)$

$(\log y + \log x) + x \phi_2(\log y$

$(D^2 + 4D'^2 - 4DD' + 2D')$

$\cdot\cdot n-2D') +$

$-D+D'$

case of

$= \frac{x^r}{b^n}$

$\phi(x$

$x\ xy$

$\log y$

$+ e$

$\log$

u

(21)

If $x_2 = 832$, $z_2 = \frac{x_2 - \mu}{\sigma} = \frac{832-750}{50} = 1.64$

$P(x_2 > 832) = P(z_2 > 1.64)$

$= 0.5 - P(0 < z < 1.64)$

$= 0.5 - 0.4495$ ——→ 2 Mark.

$= 0.0505$

Required Percentage of Persons having income exceeding Rs 680 = 94.95% = 95% (approx).

i) Let x be lowest income bar among the richest 100 persons

$z = 2.33$

$\frac{x-\mu}{\sigma} = 2.33$

$\frac{x-750}{50} = 2.33$

$x = 866.5$

2 Mark

t_{cal} $0.\cancel{025}$ 05

Null Hypothesis is accepted.

There is no significant difference between their average. → 2 Mark

(b) Null Hypothesis: There is no association between area and Pollution Index

Area	High	Low	Total
Urban	150	350	500
Rural	250	350	600
→	400	700	1100

$$E_{11} = \frac{500 \times 400}{1100} = 181.818$$

$$E_{12} = \frac{500 \times 700}{1100} = 318.18$$

$$E_{21} = \frac{700 \times 500}{1100} = 318.18$$

$$E_{22} = \frac{700 \times 600}{1100} = 381.818$$

→ 3 Mark

Chi square table

O_i	E_i	$(O_i - E_i)^2 / E_i$
150	181.818	-5.56812
350	318.18	3.182200
250	318.18	-14.6096
350	381.818	-2.6574
		-19.64692

→ 2 Mark

$\log y + \log x) + x\,\phi_2(\log y - \log x)$

$(D^2 - D^2 - 4DD' + 4D'^2 - 4D'$

$(D^2 + \dots D'^2 - 4DD' + 2D'$

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 19.64$$

Degree of freedom $= (2-1)(2-1) = 1$

$\chi^2_{cal} = 7.84$

$\chi^2_{tab} > \chi^2_{cal}$ ⇒ Null Hypothesis is Rejected

→ 2 Mark