PAPER ID-311835

Roll No: | | | | | | | | | | | | | |

BTECH
(SEM III) THEORY EXAMINATION 2025-26
MATHEMATICS-IV

TIME: 3 HRS **M.MARKS: 70**

Note: Attempt all Sections. In case of any missing data; choose suitably.
नोट: सभी प्रश्नों को हल करें। लापता डेटा को उपयुक्त मान लें।

SECTION A

1. Attempt *all* questions in brief. **02 x 7 = 14**

Q no.	Question	CO	Level
a.	What is a linear partial differential equation of first order? Write its general form. प्रथम क्रम का रैखिक आंशिक अवकल समीकरण क्या है? इसका सामान्य रूप लिखिए।	1	K1
b.	Write the one-dimensional heat equation and mention the physical meaning of each term. एक-आयामी ऊष्मा समीकरण लिखिए तथा इसके प्रत्येक पद का भौतिक अर्थ बताइए।	2	K2
c.	Define the coefficient of skewness based on moments. आघूर्णों के आधार पर विषमता गुणांक को परिभाषित कीजिए।	3	K1
d.	State Charpit's method and mention when it is used. चार्पिट की विधि को बताइए तथा यह कब प्रयुक्त की जाती है?	1	K3
e.	What is a control chart? Name any two types of control charts. नियंत्रण चार्ट क्या है? किसी भी दो प्रकार के नियंत्रण चार्ट के नाम लिखिए।	5	K2
f.	Define a probability density function (p.d.f.) and state its two properties. प्रायिकता घनत्व फलन को परिभाषित कीजिए तथा इसके दो गुण लिखिए।	4	K3
g.	What is meant by regression line of Y on X? पर Y की प्रतिगमन रेखा (Regression line of Y on X) से क्या तात्पर्य है?	3	K1

SECTION B

2. Attempt any *three* of the following: **07 x 3 = 21**

a.	Solve the linear partial differential equation $(y+z)\frac{\partial z}{\partial x}+(x+z)\frac{\partial z}{\partial y}=x+y$ using Lagrange's method. लाग्रांज की विधि का उपयोग करके निम्नलिखित रैखिक आंशिक अवकल समीकरण को हल कीजिए: $(y+z)\frac{\partial z}{\partial x}+(x+z)\frac{\partial z}{\partial y}=x+y$.	1	K5
b.	Solve the one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}, 0<x<L, t>0$ subject to the boundary conditions $u(0,t)=0, u(L,t)=0$ and the initial condition $u(x,0)=x(L-x)$ using the method of separation of variables. चर पृथक्करण विधि का उपयोग करके निम्नलिखित एक-आयामी ऊष्मा समीकरण को हल कीजिए: $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}, 0<x<L, t>0$ सीमा शर्तें : $u(0,t)=0, u(L,t)=0$ प्रारम्भिक शर्त (Initial Condition): $u(x,0)=x(L-x)$	2	K4
c.	The following data relate to variables X and Y: निम्नलिखित आँकड़े चर X तथा Y से संबंधित हैं:	3	K3

X	2	4	6	8	10
Y	5	9	13	17	21

Find the correlation coefficient and obtain the regression lines of Y on X and X on Y.
सह-संबंध गुणांक ज्ञात करें और Y पर X तथा X पर Y की प्रतिगमन रेखाएँ प्राप्त करें।

CamScanner

BTECH
(SEM III) THEORY EXAMINATION 2025-26
MATHEMATICS-IV

TIME: 3 HRS **M.MARKS: 70**

d.	The lifetime of an electric bulb is normally distributed with mean 1200 hours and standard deviation 100 hours. 1. Find the probability that a bulb lasts (a) more than 1300 hours, (b) between 1100 and 1250 hours. 2. Out of 2000 bulbs, how many are expected to last more than 1400 hours? (Standard normal table values may be assumed.) एक विद्युत बल्ब का जीवनकाल सामान्य वितरण का पालन करता है, जिसका माध्य 1200 घंटे और मानक विचलन 100 घंटे है। 1. यह ज्ञात कीजिए कि किसी बल्ब के जलने की संभावना है: (a)1300 घंटे से अधिक तक चलने की, (b) 1100 और 1250 घंटों के बीच चलने की। 2. 2000 बल्बों में से कितने बल्बों से अपेक्षा की जा सकती है कि वे 1400 घंटे से अधिक चलेंगे? (मानक सामान्य तालिका के मान मान लिए जा सकते हैं।)	4	K4
e.	Two independent random samples from two normal populations gave the following results: दो स्वतंत्र यादृच्छिक नमूनों को दो सामान्य जनसंख्या से लिया गया, जिनके परिणाम निम्नलिखित प्राप्त हुए:	5	K5

Sample(नमूना)	Size(आकार)	Mean(माध्य)	Standard Deviation(मानक विचलन)
I	10	52	6
II	12	47	5

Test whether the difference between the sample means is significant at 5% level of significance. Assume population variances are equal. परीक्षण कीजिए कि क्या नमूना माध्यों के बीच का अंतर 5% महत्व स्तर पर महत्वपूर्ण है। मान लीजिए कि जनसंख्या विचरण समान हैं।

SECTION C

3. Attempt any *one* part of the following: **07 x 1 = 07**

a.	Solve the non-linear partial differential equation $p^2 + q^2 = z$ Where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ using Charpit's method. चार्पिट की पद्धति का उपयोग करके निम्नलिखित गैर-रैखिक आंशिक अवकल समीकरण हल कीजिए: $p^2 + q^2 = z$ Where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$	1	K3
b.	Solve the partial differential equation. आंशिक अवकल समीकरण हल कीजिए $(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$	1	K5

4. Attempt any *one* part of the following: **07 x 1 = 07**

a.	Solve the wave equation तरंग समीकरण हल कीजिए $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < L$ subject to के अंतर्गत $u(0,t) = 0, u(L,t) = 0, u(x,0) = \sin \frac{\pi x}{L}, \frac{\partial u}{\partial t}(x,0) = 0.$	2	K2

CamScanner

b.	Using the Fourier transform method, solve the partial differential equation फूरियर रूपांतरण विधि का उपयोग करके आंशिक अवकल समीकरण हल कीजिए $$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t > 0$$ subject to the initial condition प्रारंभिक शर्त के अनुसार $u(x,0) = e^{-a\|x\|}$.	2	K4

5. Attempt any *one* part of the following: **07 x 1 = 07**

a.	The following data give the frequency distribution of a variable X: निम्नलिखित डेटा किसी चर X का आवृत्ति वितरण दर्शाते हैं: Find the first four moments about the mean and hence determine the coefficients of skewness and kurtosis. माध्य के बारे में पहले चार पलों को ज्ञात कीजिए और फिर इसके आधार पर विक्षेपता और गोलाई के गुणांक निर्धारित कीजिए।	3	K5

X	10	20	30	40	50
f	3	7	12	8	4

b.	Fit a parabola of the form निम्न रूप का एक परवलय फिट कीजिए $$y = a + bx + cx^2$$ to the data डेटा के अनुसार:	3	K3

x	0	1	2	3	4
y	1	2	5	10	17

6. Attempt any *one* part of the following: **07 x 1 = 07**

a.	The number of typing errors per page in a book follows a Poisson distribution with mean 2. 1. Find the probability that a randomly chosen page has (a) no error (b) at most two errors. 2. If a book has 500 pages, estimate the expected number of pages with exactly three errors. किसी पुस्तक में प्रति पृष्ठ टाइपिंग त्रुटियों की संख्या एक पॉइसन वितरण का पालन करती है, जिसका माध्य 2 है। 1. ज्ञात कीजिए कि किसी यादृच्छिक रूप से चुने गए पृष्ठ पर: (a) कोई त्रुटि नहीं होने की संभावना, (b) अधिकतम दो त्रुटियाँ होने की संभावना। 2. यदि किसी पुस्तक में 500 पृष्ठ हैं, तो अनुमान लगाइए कि कितने पृष्ठों पर ठीक तीन त्रुटियाँ होने की संभावना है।	4	K4
b.	A discrete random variable X has the following probability distribution: एक विविक्त यादृच्छिक चर X का निम्नलिखित प्रायिकता वितरण है Find the value of k and determine the **mean** and **variance** of X ("k का मान ज्ञात कीजिए और X का माध्य तथा विचरण निर्धारित कीजिए।)	4	

x	0	1	2	3	4
$P(X = x)$	k	$2k$	$3k$	$2k$	k

CamScanner

PAPER ID-311835

Roll No: | | | | | | | | | | | | | |

BTECH
(SEM III) THEORY EXAMINATION 2025-26
MATHEMATICS-IV

TIME: 3 HRS **M.MARKS: 70**

7. Attempt any *one* part of the following: **07 x 1 = 07**

a. The following table shows the distribution of students according to gender and preference of subject
निम्नलिखित तालिका छात्रों का लिंग और विषय की प्राथमिकता के अनुसार वितरण दर्शाती है: — 5 — K3

Gender(लिंग)	Science (विज्ञान)	Arts(कला)	Commerce (वाणिज्य)	Total (कुल)
Male (पुरुष)	50	30	20	100
Female (महिला)	40	50	30	120
Total (कुल)	90	80	50	220

Test at 5% level of significance whether gender and subject preference are independent
5% महत्व स्तर पर यह परीक्षण कीजिए कि क्या लिंग और विषय की प्राथमिकता स्वतंत्र हैं।

b. Five samples, each of size 4, were taken from a production process. The sample means and ranges are given below : — 5 — K5

उत्पादन प्रक्रिया से प्रत्येक का आकार 4 वाले पांच नमूने लिए गए, नमूना माध्य और फैलाव निम्नलिखित दिए गए हैं:

Sample(नमूना)	Mean (माध्य)($\bar{x}$)	Range(फैलाव) (R)
1	20	4
2	22	5
3	19	3
4	21	4
5	23	6

Construct $\bar{X}$-chart and R-chart and comment on whether the process is under control.
$\bar{X}$-चार्ट और R-चार्ट बनाइए और टिप्पणी कीजिए कि क्या यह प्रक्रिया नियंत्रण के अधीन है।.
Given
दिया गया: $A_2 = 0.729$, $D_3 = 0$, $D_4 = 2.282$.

CamScanner

SOLUTION BAS303

Section A

1. **(a)** The general form of a first-order linear partial differential equation is:

$$P(x,y)\frac{\partial z}{\partial x} + Q(x,y)\frac{\partial z}{\partial y} = R(x,y,z)$$

(b) The one-dimensional heat equation is

$$\boxed{\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}}$$

(c) The coefficient of skewness based on moments is defined as

$$\boxed{\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}}}$$

(d) For a first-order non-linear PDE of the form

$$F(x,y,z,p,q) = 0, \text{where } p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y},$$

Charpit's method consists of forming and solving the Charpit's auxiliary equations:

$$\frac{dx}{-F_p} = \frac{dy}{-F_q} = \frac{dz}{pF_p + qF_q} = \frac{dp}{F_x + pF_z} = \frac{dq}{F_y + qF_z}.$$

(e) A control chart is a graphical tool used in Statistical Quality Control (SQC) to determine whether a production process is statistically under control or not by plotting sample statistics against time with control limits. E.g p-chart, np-chart, c-chart.

(f) A probability density function (p.d.f.) of a continuous random variable X is a function $f(x)$ such that the probability that X lies in the interval $[a, b]$ is given by

$$P(a \le X \le b) = \int_a^b f(x)\,dx.$$

Non-negativity and Total probability equals one.

(g) The regression line of Y on X is the straight line that gives the best estimate (prediction) of the value of the dependent variable Y for a given value of the independent variable X.

Section B

2. **(a)** Write the auxiliary equations $\frac{dx}{y+z} = \frac{dy}{x+z} = \frac{dz}{x+y}$

Find the first integral, Choose multipliers $1, -1, 0$:

$$dx - dy = 0$$

Integrating,

$$x - y = C_1$$

CamScanner

Find the second integral Choose multipliers $1,1,-1$:

$$dx + dy - dz = 0$$

Integrating,

$$x + y - z = C_2$$

The general solution is

$$F(x - y,\ x + y - z) = 0$$

or equivalently,

$$x + y - z = \phi(x - y)$$

2 **(b)** Assume a separable solution

$$u(x,t) = X(x)T(t).$$

Substitute into the heat equation:

$$X(x)\frac{dT}{dt} = c^2 T(t)\frac{d^2X}{dx^2}.$$

Divide by c^2XT:

$$\frac{1}{c^2}\frac{1}{T}\frac{dT}{dt} = \frac{1}{X}\frac{d^2X}{dx^2} = -\lambda.$$

Solve the spatial equation

Solve the time equation

$$\frac{dT}{dt} + c^2\lambda_n T = 0.$$

$$T_n(t) = e^{-c^2\left(\frac{n\pi}{L}\right)^2 t}.$$

Form the general solution

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}\, e^{-c^2\left(\frac{n\pi}{L}\right)^2 t}.$$

Apply the initial condition

$$u(x,0) = x(L - x) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}.$$

The Fourier sine coefficients are

$$A_n = \frac{2}{L}\int_0^L x(L - x)\sin\frac{n\pi x}{L}\, dx.$$

Evaluating the integral,

$$A_n = \frac{8L^2}{n^3\pi^3}(1 - (-1)^n).$$

Thus, $A_n = 0$ for even n, and only odd terms remain.

CamScanner

Final Solution $\boxed{u(x,t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{8L^2}{n^3\pi^3} \sin\frac{n\pi x}{L}\, e^{-c^2\left(\frac{n\pi}{L}\right)^2 t}}$

2 **(c)** Compute means

$$\bar{x} = 6, \bar{y} = 13$$

Compute deviations

Let $dx = x - \bar{x}$, $dy = y - \bar{y}$

$$\sum dxdy = 80, \sum dx^2 = 40, \sum dy^2 = 160$$

Correlation coefficient

$$r = \frac{\sum dxdy}{\sqrt{\sum dx^2 \sum dy^2}} = \frac{80}{\sqrt{6400}} = 1$$

Regression equations

***Y* on *X*:**

$$y - \bar{y} = \frac{\sum dxdy}{\sum dx^2}(x - \bar{x}) \Rightarrow y = 2x + 1$$

***X* on *Y*:**

$$x - \bar{x} = \frac{\sum dxdy}{\sum dy^2}(y - \bar{y}) \Rightarrow x = \frac{1}{2}y - 0.5$$

2 **(d)** Standardization

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{1300 - 1200}{100} = 1$$

$$P(Z > 1) = 0.1587$$

$$Z_1 = \frac{1100 - 1200}{100} = -1 \; Z_2 = \frac{1250 - 1200}{100} = 0.5$$

$$P(-1 < Z < 0.5) = 0.3413 + 0.1915 = 0.5328$$

Expected bulbs lasting more than 1400 hours

$$Z = \frac{1400 - 1200}{100} = 2$$

$$P(Z > 2) = 0.0228$$

Expected bulbs $= 2000 \times 0.0228 = 46$

P(X>1300)=0.1587, P(1100<X<1250)=0.5328, Bulbs≈46

2 **(e)** State hypotheses

Pooled variance

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{9(36) + 11(25)}{20} = \frac{324 + 275}{20} = 29.95$$

$$s_p = \sqrt{29.95} = 5.47$$

Compute t-statistic

CamScanner

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$
$$= \frac{52 - 47}{5.47\sqrt{\frac{1}{10} + \frac{1}{12}}} = 2.21$$

Critical value

$$\text{d.f.} = 10 + 12 - 2 = 20$$
$$t_{0.05,20} = 2.086$$

Decision
Since

$$t_{cal} = 2.21 > t_{tab} = 2.086$$

Reject H_0

Section C

3. **(a)**

Given,

$$F(x, y, z, p, q) = p^2 + q^2 - z = 0$$

$F_x = 0, F_y = 0, F_z = -1$

Charpit's equations

$$\frac{dx}{2p} = \frac{dy}{2q} = \frac{dz}{2p^2 + 2q^2} = \frac{dp}{p} = \frac{dq}{q}$$

Solve for p and q

From

$$\frac{dp}{p} = \frac{dq}{q} \Rightarrow \ln p - \ln q = C \Rightarrow \frac{p}{q} = a$$

Thus, $p = aq$
Substitute in given PDE

$$a^2q^2 + q^2 = z \Rightarrow q^2(1 + a^2) = z \Rightarrow q = \sqrt{\frac{z}{1 + a^2}}$$

Since

$$\frac{\partial z}{\partial y} = q = \sqrt{\frac{z}{1 + a^2}}$$
$$\int \frac{dz}{\sqrt{z}} = \int \frac{dy}{\sqrt{1 + a^2}}$$

CamScanner

$$2\sqrt{z} = \frac{y}{\sqrt{1+a^2}} + b$$

$$z = \left(\frac{y}{2\sqrt{1+a^2}} + b\right)^2$$

3 **(b)** Auxiliary equation:

$$m^2 - 4mn + 4n^2 = 0$$
$$(m-2n)^2 = 0 \Rightarrow m = 2n \text{(repeated root)}$$

Thus, C.F. $= \phi_1(y+2x) + x\,\phi_2(y+2x)$
Particular Integral (P.I.)

Given RHS: e^{2x+y}

$$P.I. = \frac{e^{2x+y}}{(2)^2 - 4(2)(1) + 4(1)^2}$$
$$= \frac{e^{2x+y}}{4-8+4} = \frac{e^{2x+y}}{0}$$

Since denominator is zero, multiply by x:

$$P.I. = x \cdot \frac{e^{2x+y}}{(2D - 4D' + 4D'^2)'}$$

Finally,

$$P.I. = \frac{xe^{2x+y}}{4}$$

General Solution

$$\boxed{z = \phi_1(y+2x) + x\,\phi_2(y+2x) + \frac{xe^{2x+y}}{4}}$$

4. (a) Assume separable solution

$$u(x,t) = X(x)T(t)$$

Substitute:

$$X\frac{d^2T}{dt^2} = c^2T\frac{d^2X}{dx^2} \Rightarrow \frac{1}{c^2}\frac{T''}{T} = \frac{X''}{X} = -\lambda$$

$$X'' + \lambda X = 0$$

Boundary conditions give:

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2$$
$$X_n(x) = \sin\frac{n\pi x}{L}$$

CamScanner

Solve time equation

$$T_n'' + c^2\lambda_n T_n = 0$$
$$T_n(t) = A_n \cos\frac{n\pi ct}{L} + B_n \sin\frac{n\pi ct}{L}$$

Apply initial conditions
Initial displacement:

$$u(x,0) = \sin\frac{\pi x}{L} \Rightarrow A_1 = 1,\ A_{n\neq 1} = 0$$

Initial velocity:

$$\frac{\partial u}{\partial t}(x,0) = 0 \Rightarrow B_n = 0$$

Final solution

$$\boxed{u(x,t) = \sin\frac{\pi x}{L}\cos\frac{\pi ct}{L}}$$

4 (b) Take Fourier transform
Let

$$\mathcal{F}\{u(x,t)\} = U(\omega,t)$$

Using properties:

$$\mathcal{F}\left\{\frac{\partial^2 u}{\partial x^2}\right\} = -\omega^2 U(\omega,t)$$

Transforming PDE:

$$\frac{\partial U}{\partial t} = -k\omega^2 U$$

Solve transformed ODE

$$\frac{dU}{dt} = -k\omega^2 U$$
$$U(\omega,t) = C(\omega)e^{-k\omega^2 t}$$

Transform initial condition

$$\mathcal{F}\{e^{-a|x|}\} = \frac{2a}{a^2+\omega^2}$$

Thus,

$$C(\omega) = \frac{2a}{a^2+\omega^2}$$

Inverse Fourier transform

$$u(x,t) = \mathcal{F}^{-1}\left\{\frac{2a}{a^2+\omega^2}e^{-k\omega^2 t}\right\}$$

Evaluating,

$$\boxed{u(x,t) = \frac{a}{\sqrt{\pi kt}}\int_0^\infty e^{-k\omega^2 t}\cos(\omega x)\frac{d\omega}{a^2+\omega^2}}$$

5. **(a)** Compute mean

CamScanner

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\sum f = 34, \sum fx = 1020 \Rightarrow \bar{x} = 30$$

Compute deviations

Let $d = x - \bar{x}$

Moments about the mean

$$\mu_2 = \frac{4300}{34} = 126.47$$

$$\mu_3 = \frac{9000}{34} = 264.71$$

$$\mu_4 = \frac{1{,}270{,}000}{34} = 37352.94$$

Skewness $\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}} = 0.186$

Kurtosis $\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3 = -0.67$

5 **(b)**

Normal equations

$$\sum y = na + b\sum x + c\sum x^2$$
$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$
$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Compute sums

$$\sum y = 35, \sum x = 10, \sum x^2 = 30$$
$$\sum x^3 = 100, \sum x^4 = 354$$
$$\sum xy = 108, \sum x^2 y = 360$$

Solve equations

$$5a + 10b + 30c = 35$$
$$10a + 30b + 100c = 108$$
$$30a + 100b + 354c = 360$$

Solving.

$$a = 1, b = 0, c = 1$$

Fitted parabola

$$\boxed{y = 1 + x^2}$$

6. **(a)** Poisson formula

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ where } \lambda = 2$$

Probability of no error

$$P(X = 0) = e^{-2} = 0.1353$$

At most two errors

$$P(X \leq 2) = P(0) + P(1) + P(2)$$
$$= e^{-2}\left(1 + 2 + \frac{2^2}{2!}\right) = e^{-2}(1 + 2 + 2) = 5e^{-2}$$

CamScanner

$$= 0.6767$$

Expected pages with exactly 3 errors

$$P(X = 3) = \frac{e^{-2}2^3}{3!} = \frac{4}{3}e^{-2}$$

$$\text{Expected number} = 500 \times \frac{4}{3}e^{-2} \approx 90$$

P(0)=0.1353, P(X≤2)=0.6767, Pages≈90

(b)

Find k

$$\sum P(X = x) = 1$$

$$k + 2k + 3k + 2k + k = 9k = 1 \Rightarrow k = \frac{1}{9}$$

Mean $E(X)$

$$E(X) = \sum xP(X = x)$$

$$= \frac{1}{9}(0 + 2 + 6 + 6 + 4) = \frac{18}{9} = 2$$

Compute $E(X^2)$

$$E(X^2) = \sum x^2 P(X = x)$$

$$= \frac{1}{9}(0 + 2 + 12 + 18 + 16) = \frac{48}{9} = \frac{16}{3}$$

Variance

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \frac{16}{3} - 4 = \frac{4}{3}$$

$$\boxed{E(X) = 2, \text{Var}(X) = \frac{4}{3}}$$

7. **(a)** Hypotheses

H_0:Gender and subject preference are independent

H_1:They are not independent

Expected frequencies

$$E_{ij} = \frac{(\text{Row total})(\text{Column total})}{\text{Grand total}}$$

Example:

$$E_{11} = \frac{100 \times 90}{220} = 40.91$$

Similarly, expected frequencies are calculated for all cells.

Chi-square statistic

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$\chi^2_{cal} = 8.12$$

Critical value

$$\text{d.f.} = (r - 1)(c - 1) = (2 - 1)(3 - 1) = 2$$

CamScanner

$$\chi^2_{0.05,2} = 5.991$$

Decision

$$\chi^2_{cal} > \chi^2_{tab} \Rightarrow \boxed{\text{Reject } H_0}$$

There is a significant association between gender and subject preference.

(b) Compute averages

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{5} = \frac{105}{5} = 21$$

$$\bar{R} = \frac{\sum R}{5} = \frac{22}{5} = 4.4$$

$\bar{X}$-chart limits

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 21 + 0.729(4.4) = 24.21$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 21 - 3.21 = 17.79$$

R-chart limits

$$UCL = D_4\bar{R} = 2.282(4.4) = 10.04$$

$$LCL = D_3\bar{R} = 0$$

Interpretation

All sample means and ranges lie within control limits.

The production process is statistically under control

CamScanner